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FERMILAB-Pub-86/71-A
May 1986

General Constraints on the Age and Chemical Evolution of the Galaxy

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ABSTRACT

The formalism of Schramm and Wasserburg (1970) for determining the mean age of the elements is extended. Model-independent constraints (constraints that are independent of a specific form for the effective nucleosynthesis rate and Galactic chemical evolution over time) are derived on the first four terms in the expansion giving the mean age of the elements, and from these constraints limits are derived on the total duration of nucleosynthesis. These limits require only input of the Schramm-Wasserburg parameter Δ^{max} and of the ratio of the mean time for formation of the elements to the total duration of nucleosynthesis, t_ν/T. The former quantity is a function of nuclear input parameters. Limits on the latter are obtained from constraints on the relative rate of nucleosynthesis derived from the $^{232}Th/^{238}U$, $^{235}U/^{238}U$, and shorter-lived chronometric pairs. Because ^{187}Re may decay faster in hot stars than in interstellar space, its effective lifetime may be less than the laboratory value; thus, using the laboratory decay rate gives an upper limit on Δ^{max} for the ^{187}Re/^{187}Os pair, which gives an upper limit on the duration of nucleosynthesis. A lower limit on Δ^{max} can be determined from the ^{232}Th/^{238}U pair, which then yields a lower limit on the duration of nucleosynthesis. The results found are that *the effective nucleosynthesis rate was relatively constant over most of the duration of nucleosynthesis and that* $0.43 \lesssim t_\nu/T \lesssim 0.59$. From these constraints on t_ν/T, a nearly model-independent range for T_{Gal}, the age of the Galaxy, is obtained: $8.7\ \mathrm{Gyr} \lesssim T_{Gal} \lesssim 28.1\ \mathrm{Gyr}$. Improvements in nuclear and meteoritic data could lead to a dramatic narrowing of this model-independent range in the near future. Detailed Galaxy evolution models give a far narrower range on the age, but the results depend on assumptions about the specific form of the effective nucleosynthesis rate.

Subject headings: abundances - galaxies: evolution - galaxies: Milky Way - nucleosynthesis

I. Introduction

Nucleocosmochronolgy employs knowledge of abundance and production ratios of radioactive nuclides and of the chemical evolution of the Galaxy to obtain information about timescales over which the solar system elements were synthesized. The field may be said to have begun with Rutherford's (1929) attempt to infer the duration of the time for synthesis of uranium isotopes. Model-dependent cosmochronology, which *assumes* models of Galactic chemical evolution, was developed by Burbidge et al. (1957) who considered sudden synthesis and constant synthesis models and by Fowler and Hoyle (1960) who used exponentially decaying synthesis models. These authors relied on the ^{232}Th/^{238}U and ^{235}U/^{238}U chronometric pairs. Later Wasserburg et al. (1969), Hohenburg (1969), Fowler (1972), and Kohman (1972) included the shorter-lived chronometric pairs ^{244}Pu/^{238}U and ^{129}I/^{127}I in their model-dependent studies. Also Clayton (1964) introduced the ^{187}Re/^{187}Os chronometric pair which held great promise for accurate determinations of the Galaxy's age since ^{187}Re has a 43 Gyr halflife, much greater than the suspected age of the Galaxy. The 1970's saw the introduction of more detailed Galaxy evolution models in terms of which model-dependent cosmochronology could be further studied (e.g. Talbot and Arnett 1973; Tinsley 1975, 1977, 1980; Ostriker and Thuan 1975; Hainebach and Schramm 1977). Work in model-dependent nucleocosmochronology continues in the framework of both the exponential model for nucleosynthesis (e.g. Thielemann et al. 1983a, 1983b; Thielemann 1984; Thielemann and Truran 1985) and more detailed nucleosynthesis models (e.g. Yokoi et al. 1983; Clayton 1984a, 1984b, 1984c).

Despite the considerable amount of work done on nucleocosmochronology, many uncertainties in the nuclear and meteoritic data at present preclude accurate conclusions about the Galaxy's age. For the Re/Os chronometric pair, astration may significantly increase the ^{187}Re β-decay rate over the laboratory rate

(Takahashi and Yokoi 1982; Yokoi et al. 1983), thereby greatly affecting Galaxy age determinations from this pair. Furthermore, the low lying excited nuclear state of ^{187}Re at 9.8 kev complicates the subtraction of s-process ^{187}Os from the total ^{187}Os abundance (Fowler 1973; Holmes et al. 1976; Woosley and Fowler 1979) as do possible s-process branchings in the W, Re, and Os region (Arnould 1974; Arnould et al. 1984). As the r-process only abundance of ^{187}Os is required for use of the Re/Os pair for cosmochronology, the two above uncertainties make accurate Galactic age conclusions from this pair difficult. Also, β-delayed fission and neutron emission may greatly affect the actinide production ratios (Thielemann et al. 1983a, 1983b; Meyer et al. 1985), thereby making age predictions from those pairs change drastically from earlier predictions. We discuss these nuclear data uncertainties and also meteoritic data uncertainties in somewhat greater detail in section VII of this paper.

Assumptions about the model for Galactic chemical evolution are also a source of error in age estimates for the Galaxy. If the wrong model is chosen, factor of two or so errors may find their way into age determinations. For this reason we may turn to model-independent cosmochronology, which studies those cosmochronological conclusions that may be made about the Galaxy's age without reference to a model for Galaxy nucleosynthesis. Schramm and Wasserburg (1970; hereafter SW) showed that the main thing that nucleocosmochronology can give us independent of Galactic evolution model is the mean age of the elements, which provides us with a lower limit on the Galaxy's age. We may also ask whether it is possible to obtain a model-independent upper limit to the Galaxy's age. If so, then a model-independent range will be available as a framework within which more model-dependent work may be considered.

It is the goal of this paper, then, to develop as model-independent a range for the Galaxy's age as possible that takes all nuclear and meteoritic data uncertainties into account. As the mean age of the elements provides a natural, model-independent lower limit on the Galaxy's age, our major task is to obtain a model-independent upper limit. We derive such an upper limit in section IV from an expansion of the equation giving the mean age of the elements in moments of the normalized effective nucleosynthesis rate made in section III. This limit depends only on t_ν/T, the ratio of the mean time of formation of the elements to the total duration of nucleosynthesis, and on model-independent data. In section V we show that the $^{232}Th/^{238}U$, $^{235}U/^{238}U$, and $^{244}Pu/^{238}U$ chronometric pairs can give us constraints on the relative rate of nucleosynthesis over the history of the synthesis of the solar system material. In section VI we use the results of section V to derive limits on t_ν/T which then allow us to derive nearly model-independent limits on the Galaxy's age. Section VII discusses input data and uncertainties in these data. In section VIII we present our results and in section IX our conclusions.

II. The Basic Equations and Galaxy Evolution Framework

The general equation governing the time evolution of the abundance N_i of nuclide i in the interstellar medium of the Galaxy is (SW)

$$\frac{dN_i(r,t)}{dt} = -\lambda_i N_i(r,t) + B(r,N_i,t), \tag{2.1}$$

where λ_i is the decay rate for nuclide i and $B(r, N_i, t)$ is the generalized production and destruction (that is, destruction by means other than radioactive decay) function. To be useful for nucleocosmochronology, equation (2.1) must be simplified by means of a Galactic evolution model. A general model is that of Tinsley (1975); see also Hainebach and Schramm (1977) and Symbalisty and Schramm (1981). The result of this model is that equation (2.1) is linearized:

$$\frac{dN_i(t)}{dt} = -\lambda_i N_i(t) - \omega(t) N_i(t) + P_i \psi(t). \tag{2.2}$$

$\omega(t)$ is a time-dependent parameter representing the rate of movement of metals into and out of the interstellar medium for reasons other than decay, $\psi(t)$ is the amount of mass going into stars per unit time, and P_i is the number of nuclei produced per unit mass going into stars. It should be noted that in going from equation (2.1) to equation (2.2), we have assumed that the history of material going into the solar system was spatially homogeneous and that the instantaneous recycling approximation holds.

It is now possible to solve for the abundance N_i of nuclide i at a given time by integration of equation (2.2). We do this in the context of the scenario for evolution of the material making up the solar system shown in Figure 1. In Figure 1, T is the time of the last event contributing to formation of the elements going into the solar system, Δ is the time interval between this last nucleosynthetic event and the solidification of the solid bodies in the solar system, and t_{ss} ($= 4.55 \times 10^9$yr) is the age of the solid bodies in the solar system. It is clear that, in this scenario, Δ represents a period of free decay for the elements. Since the meteoritic material is a closed system after T+Δ, however, it gives abundances at times as early as T+Δ with minimal uncertainty due to chemical fractionation. Integration of equation (2.2), then, for t=0 to t=T followed by free decay over an interval Δ yields

$$N_i(T + \Delta) = P_i \exp(-\lambda_i\Delta) \exp(-\lambda_i T - \nu(T)) \int_0^T \psi(t)\exp(\lambda_i t + \nu(t))dt, \tag{2.3}$$

where

$$\nu(t) = \int_0^t \omega(\xi)d\xi \tag{2.4}$$

and we have assumed P_i to be constant in time.

Equation (2.3) is the point of departure for the rest of this paper.

III. Expansion in Moments

The abundance $N_i(T+\Delta)$ in equation (2.3) is dependent upon the effective nucleosynthesis rate ψe^ν; therefore, information is needed about the chemical evolution over the nucleosynthesis interval T before equation (2.3) can be solved. SW have shown, however, that useful constraints on the Galaxy's age can be obtained, independent of any information on Galaxy evolution, through expansion of the equation for $N_i(T+\Delta)$ in moments of the production function about the mean age. We proceed in analogous fashion, using the effective nucleosynthesis rate ψe^ν in place of SW's $\rho(\tau)$.

We begin by defining $\phi(t)$, the normalized effective nucleosynthesis rate as

$$\phi(t) \equiv \frac{\psi e^\nu}{<\psi>T}, \tag{3.1}$$

where $<\psi>$, the average effective nucleosynthesis rate, is defined as

$$<\psi> \equiv \frac{1}{T}\int_0^T \psi e^{\nu} dt. \tag{3.2}$$

In this case, equation (2.3) becomes

$$N_i(T + \Delta) = P_i\, T <\psi> \exp(-\lambda_i\Delta) \exp(-\lambda_i T - \nu(T)) \int_0^T \phi(t) e^{\lambda_i t} dt. \tag{3.3}$$

Now, with Tinsley's (1975) definition of the mean time for formation of the elements

$$t_\nu \equiv \frac{1}{T<\psi>} \int_0^T t\psi e^{\nu} dt = \int_0^T t\phi(t) dt, \tag{3.4}$$

we can write

$$N_i(T + \Delta) = P_i\, T<\psi> \exp(-\lambda_i\Delta) \exp(-\nu(T) \exp(-\lambda_i(T - t_\nu)) \int_0^T \phi(t) e^{\lambda_i(t - t_\nu)} dt. \tag{3.5}$$

Analogously with SW, we expand equation (3.5) in moments of the normalized effective nucleosynthesis rate $\phi(t)$. This yields

$$N_i(T + \Delta) = P_i\, T<\psi> \exp(-\lambda_i\Delta)\exp(-\nu(T)) \,[1 + \delta_i], \tag{3.6}$$

where

$$\delta_i \equiv \sum_{n=2}^{\infty} \frac{\lambda_i{}^n}{n!} \mu_n \tag{3.7}$$

and μ_n, the nth moment of $\phi(t)$ about t_ν, is

$$\mu_n \equiv \int_0^T (t - t_\nu)^n \phi(t) dt. \tag{3.8}$$

If we divide our expression for $N_i(T + \Delta)$ in equation (3.6) by the corresponding one for a different nuclide, nuclide j, and take the natural logarithm, we obtain

$$T - t_\nu = \frac{\ln[(P_i/P_j)/(N_i(T+\Delta)/N_j(T+\Delta))]}{\lambda_i - \lambda_j} - \Delta + \frac{1}{\lambda_i - \lambda_j}\ln[\frac{1+\delta_i}{1+\delta_j}]. \tag{3.9}$$

With the convenient definition of the one event age Δ_{ij}^{max},

$$\Delta_{ij}^{max} \equiv \frac{\ln[(P_i/P_j)/(N_i(T+\Delta)/N_j(T+\Delta))]}{\lambda_i - \lambda_j} \equiv \frac{\ln R(i,j)}{\lambda_i - \lambda_j}, \tag{3.10}$$

we find

$$T - t_\nu = \Delta_{ij}^{max} - \Delta + \frac{1}{\lambda_i - \lambda_j}\ln[\frac{1+\delta_i}{1+\delta_j}]. \tag{3.11}$$

Equation (3.11) is exactly analogous to equation (6) of SW. The difference between the two is that $\rho(t)$ in SW lacks the factor e^{ν} contained in $\phi(t)$ (equation (3.1)). This difference shows up in the expansion of equation (3.11) to 4th order in μ,

$$T - t_\nu = \Delta_{ij}^{max} - \Delta + \frac{(\lambda_i + \lambda_j)\mu_2}{2} + \frac{(\lambda_i^2 + \lambda_i\lambda_j + \lambda_j^2)\mu_3}{6} +$$

$$+ \frac{1}{4}(\frac{\mu_4}{6} - \mu_2^2)(\frac{\lambda_i^4 - \lambda_j^4}{\lambda_i - \lambda_j}) + \cdots \tag{3.12}$$

The $\omega(t)$ dependence is now contained in the μ's as opposed to being contained in the coefficients of the μ's in SW. Also t_ν replaces SW's $<\tau>$.

Since μ_n is essentially proportional to T^n, for long-lived chronometers ($\lambda T << 1$) equation (3.11) reduces to

$$T - t_\nu \approx \Delta^{max} - \Delta, \tag{3.13}$$

which is the long-lived limit obtained by SW, with t_ν in place of $<\tau>$. Equation (3.13) makes sense because a sufficiently long-lived chronometer sees nucleosynthesis as a single event at its mean age t_ν. Clearly, then, the moment terms in equation (3.12) represent corrections to the long-lived limit due to the finite lifetimes of the chronometers in question and Galaxy evolution.

IV. Model-Independent Constraints

Calculation of $T - t_\nu$ from equation (3.12) requires knowledge of the various μ's, which in turn require knowledge of $\phi(t)$ and T. These, however, are parameters we are trying to determine. In the long-lived limit, however, equation (3.13) shows that we need no information on $\phi(t)$ or T to calculate $T - t_\nu$. This is, of course, the significance of the long-lived limit. Meyer and Schramm (1985), however, have shown that the supposedly long-lived chronometric pair $^{232}Th/^{238}U$ may not be sufficiently long-lived that equation (3.13) holds, although it probably does for $^{187}Re/^{187}Os$. Thus comparisons between their respective mean ages should be made with caution. In any case, it is clear that we must know (or at least suspect) that a chronometric pair is long-lived before we use equation (3.13). This, however, requires comparison of $T - t_\nu$ calculated from equations (3.12) and (3.13); thus, we are again confronted with the dilemma of having to know $\phi(t)$ and T before calculating them.

The common way out of this dilemma is to *assume* some $\phi(t)$ (i.e. Galaxy evolution model). The results thus obtained are clearly model-dependent. We wish to attempt a solution to the problem that is as model-independent as possible.

Before attempting to determine upper limits on T, we note that since the higher moment terms in equation (3.12) tend to increase $T - t_\nu$ over $\Delta^{max} - \Delta$ (see e.g. Meyer and Schramm 1985), $\Delta^{max} - \Delta$ provides a lower limit on T, that is,

$$T \geq \Delta^{max} - \Delta. \tag{4.1}$$

If we know t_ν/T, we can push this lower limit up to

$$T \geq \left(1 - \frac{t_\nu}{T}\right)^{-1} (\Delta^{max} - \Delta). \tag{4.2}$$

In section VI we will derive limits on t_ν/T.

Let us now consider possible upper limits on the μ's defined in equation (3.8). In the case where n is even, μ_n will be maximized for a $\phi(t)$ that has its production as far as possible from $t=t_\nu$. We are thus led to conclude that the normalized effective nucleosynthesis rate that gives the largest possible μ_n is of the form

$$\phi(t) = C\delta(t) + D\delta(t - T). \tag{4.3}$$

The coefficients C and D are determined from equation (3.4) and normalization, which yield

$$D = \frac{t_\nu}{T} \tag{4.4}$$

and

$$C = 1 - D. \tag{4.5}$$

Substitution of equation (4.3) into (3.8) and integration yield

$$\frac{\mu_n}{T^n} = (1 - D)D^n + D(1 - D)^n, \tag{4.6}$$

since n is even. Maxima of equation (4.6) are those D satisfying

$$nD^{n-1} - (n + 1)D^n + (1 - D)^n - nD(1 - D)^{n-1} = 0. \tag{4.7}$$

For n=2, D=1/2. For n=4, D=$1/2 \pm 1/\sqrt{12}$. These give

$$\frac{\mu_2}{T^2} \leq \frac{1}{4} \tag{4.8}$$

and

$$\frac{\mu_4}{T^4} \leq \frac{1}{12}. \tag{4.9}$$

In the case where n is odd, μ_n measures the asymmetry of ψe^{ν} about $t=t_{\nu}$. It is not as clear for this case what the maximizing shape of ψe^{ν} would be. In the appendix we are able to make general arguments that result in a limit

$$\frac{\mu_3}{T^3} \lesssim \frac{1}{13.1} \tag{4.10}$$

Alternatively, we may again use equation (4.3). For n odd, we find

$$\frac{\mu_n}{T^n} = (D - 1)D^n + D(1 - D)^n. \tag{4.11}$$

The maxima now occur for D satisfying

$$(n + 1)D^n - nD^{n-1} + (1 - D)^n - nD(1 - D)^n = 0. \tag{4.12}$$

For n=3, $D = 1/2 \pm 1/\sqrt{12}$, which yields

$$\frac{\mu_3}{T^3} \lesssim \frac{1}{10.4}. \tag{4.13}$$

The limit in equation (4.13) contains that in equation (4.10), and, hence we will use it.

We now substitute equations (4.8), (4.9), and (4.13) into equation (3.12) to get

$$T - t_\nu \lesssim (\Delta^{max} - \Delta) + \frac{1}{8}(\lambda_i + \lambda_j)T^2 + \frac{5}{312}(\lambda_i^2 + \lambda_i\lambda_j + \lambda_j^2)T^3 +$$

$$+ \frac{1}{288}(\frac{\lambda_i^4 - \lambda_j^4}{\lambda_i - \lambda_j})T^4 + \cdots, \qquad (4.14)$$

where we have used the fact that $\mu_4/6 \geq \mu_4/6 - \mu_2^2$. Equation (4.14) can be solved iteratively if the chronometers allow the sum on the right hand side to converge sufficiently rapidly and if t_ν/T can be estimated. This is best done by writing

$$T - t_\nu = (\Delta^{max} - \Delta)(1 + \epsilon). \qquad (4.15)$$

ϵ is the sum of all of the higher moment terms in the expansion in equation (4.14). Substitution of equation (4.15) into equation (4.14) yields

$$\epsilon \lesssim \frac{1}{8}(1 - \frac{t_\nu}{T})^{-2}(\lambda_i + \lambda_j)(\Delta^{max} - \Delta)(1 + \epsilon)^2 + \frac{5}{312}(1 - \frac{t_\nu}{T})^{-3}(\lambda_i^2 + \lambda_i\lambda_j + \lambda_j^2)(\Delta^{max} - \Delta)^2(1 + \epsilon)^3 +$$

$$+ \frac{1}{288}(1 - \frac{t_\nu}{T})^{-4}(\frac{\lambda_i^4 - \lambda_j^4}{\lambda_i - \lambda_j})(\Delta^{max} - \Delta)^3(1 + \epsilon)^4 + \cdots \qquad (4.16)$$

An upper limit on T is thus

$$T \lesssim (1 - \frac{t_\nu}{T})^{-1}(\Delta^{max} - \Delta)(1 + \epsilon). \qquad (4.17)$$

As mentioned above, in section VI we will see that limits on t_ν/T can be estimated.

V. Galaxy Evolution Constraints

In the preceding sections, we have seen that we can get upper limits on T provided we know t_ν/T. In this section and the next, we derive a means of estimating t_ν/T.

It is possible that certain radionuclides are sufficiently short-lived that essentially all of the nuclei produced prior to some time $t=\bar{\tau}$ make no contribution to the abundance of that nuclide at $t=T+\Delta$. Assume this is true for nuclide i. In this case, we can write equation (2.3) as

$$N_i(T+\Delta) = P_i e^{-\lambda_i \Delta} e^{-\nu(T)} \int_{\bar{\tau}_i}^{T} \psi e^{\nu} e^{-\lambda(T-t)} dt, \tag{5.1}$$

where $\bar{\tau}_i$ has replaced zero as the lower limit on the integral. We now define an average nucleosynthesis rate $<\psi>_{\bar{\tau}_i,i}$ over the interval $\bar{\tau}_i \leq t \leq T$:

$$<\psi>_{\bar{\tau}_i,i} \equiv \frac{\int_{\bar{\tau}_i}^{T} \psi e^{\nu} e^{-\lambda_i(T-t)} dt}{\int_{\bar{\tau}_i}^{T} e^{-\lambda_i(T-t)} dt}. \tag{5.2}$$

This average rate is exactly analogous to the average rate defined in equation (14) of Reeves and Johns (1976). The difference is that the integrals in equation (5.2) above begin at $t=\bar{\tau}_i$ instead of $t=0$. By varying $\bar{\tau}_i$, we are able to average ψe^{ν} over a variety of intervals.

Substitution of equation (5.2) into equation (5.1) and integration yield

$$N_i(T+\Delta) = \frac{P_i e^{-\lambda_i \Delta} e^{-\nu(T)} <\psi>_{\bar{\tau}_i,i} (1 - e^{-\lambda_i(T-\bar{\tau}_i)})}{\lambda_i}. \tag{5.3}$$

We can express $T - \bar{\tau}_i$ as some number α_i of half-lifes of nuclide i, that is,

$$T - \bar{\tau}_i = \alpha_i \tau_i. \tag{5.4}$$

We are thus able to write

$$N_i(T+\Delta) = \frac{P_i e^{-\lambda_i \Delta} e^{-\nu(T)} <\psi>_{\bar{\tau}_i,i}}{\lambda_i} (1 - (\frac{1}{2})^{\alpha_i}) \tag{5.5}$$

by the definition of the half-life. In the case where we choose $\bar{\tau}_i = 0$, however, it is more convenient to stay with equation (5.3) in the form

$$N_i(T+\Delta) = \frac{P_i e^{-\lambda_i \Delta} e^{-\nu(T)} <\psi>_{0,i}}{\lambda_i}(1 - e^{-\lambda_i T}), \tag{5.6}$$

which might be called the Reeves and Johns form.

If we have two nuclides i and j, with two different τ_i and τ_j, respectively, we can choose $\bar{\tau}_i$ and $\bar{\tau}_j$ such that we average over the same number of half-lifes for each, that is, $\alpha_i = \alpha_j$. In this case, we find from equation (5.5) that

$$\frac{<\psi>_{\bar{\tau}_i,i}}{<\psi>_{\bar{\tau}_j,j}} = \frac{e^{(\lambda_i - \lambda_j)\Delta}}{R(i,j)} \frac{\lambda_i}{\lambda_j}, \tag{5.7}$$

where R(i,j) is the standard nucleocosmochronology input parameter defined in equation (3.10).

The significance of equation (5.7) is two-fold. First, it is independent of T. Second, it is independent of the number of half-lifes we average over since we average over the same number of half-lifes for i and j.

We now make the convention that nuclide i is shorter-lived than nuclide j, that is, $\tau_i < \tau_j$. The interpretation of equation (5.7) is then simple. If $<\psi>_{\bar{\tau}_i,i}/<\psi>_{\bar{\tau}_j,j}$ is approximately one, then ψe^{ν} was roughly steady over a few times τ_i; if $<\psi>_{\bar{\tau}_i,i}/<\psi>_{\bar{\tau}_j,j} <1$, then ψe^{ν} was generally falling over a few times τ_i; and if $<\psi>_{\bar{\tau}_i,i}/<\psi>_{\bar{\tau}_j,j} >1$, then ψe^{ν} was generally rising over a few times τ_i.

In all of the above we have assumed that essentially all of the nuclei produced prior to $t = \bar{\tau}$ have decayed by $t = T$. This is not the case for the long-lived nuclei, which may have half-lifes comparable to or even greater than T.

Clearly, in this case, we must use equation (5.6) for nuclide j, which yields

$$\frac{<\psi>_{\bar{\tau}_i,I}}{<\psi>_{0J}} = \frac{e^{(\lambda_i-\lambda_j)\Delta}}{R(i,j)} \frac{(1-e^{-\lambda_j T})}{(1-(\frac{1}{2})^{\alpha_i})} \frac{\lambda_I}{\lambda_J}. \tag{5.8}$$

This depends on both α_I and T. If we allow α_I to grow, eventually $\alpha_i\tau_i$ will be greater than T, and we will then have to use

$$\frac{<\psi>_{0,I}}{<\psi>_{0J}} = \frac{e^{(\lambda_i-\lambda_j)\Delta}}{R(i,j)} \frac{(1-e^{-\lambda_j T})}{(1-e^{-\lambda_i T})} \frac{\lambda_I}{\lambda_J}, \tag{5.9}$$

which is equation (15) of Reeves and Johns (1976) multiplied by a factor $e^{(\lambda_i-\lambda_j)\Delta}$.

If we wish to compare short- and long-lived chronometers, we are forced to use equation (5.8). Equation (5.7) is still useful, however, since it gives an upper limit on $<\psi>_{\bar{\tau}_i,I}/<\psi>_{\bar{\tau}_j J}$, independently of T. This is clear since the largest $(1-e^{-\lambda_j T})/(1-e^{-\lambda_i T})$ can be is unity. Correspondingly, equation (5.9) gives the smallest possible value, for a given T, for the ratio $<\psi>_{\bar{\tau}_i,I}/<\psi>_{\bar{\tau}_j J}$ since equation (5.9) is the limiting case of equation (5.8) for large α_I.

VI. Constraints on t_ν/T

With constraints on ψe^ν available over a variety of time intervals, we now use this information to constrain t_ν/T and to provide a means of solving for T directly.

i) Galaxy Evolution Constraints

We define the parameter r(i,j) as the ratio of $<\psi>_i/<\psi>_j$, where $<\psi>_I/<\psi>_J$ is given by equation (5.7), (5.8), or (5.9). Let us now assume a set of m chronometers. We label our longest-lived chronometer by i=1, the next longest-lived by i=2, and so on to the shortest-lived labelled i=m. It is now possible to develop Galaxy evolution constraints.

To begin, let us assume we have two chronometers. r(2,1) is the ratio $<\psi>_{\bar{\tau}_2,2}/<\psi>_{\bar{\tau}_1,1}$. If we average ψe^{ν} for nuclide 1 over all of T, then $\alpha_1 = T/\tau_1$. Now let us take $\alpha_2 = \alpha_1$, then r(2,1) represents approximately the ratio of ψe^{ν} over α_1 half- lifes of nuclide 2 to ψe^{ν} over α_1 half-lifes of nuclide 1. We thus assume $\psi e^{\nu} = r(1,1)=1$ (in some arbitrary units which do not matter since we will eventually normalize to get $\phi(t)$) from t=0 to $t=t_1 \equiv \alpha_1(\tau_1 - \tau_2)$ and $\psi e^{\nu} = r(2,1)$ from $t=t_1$ to t=T.

The above picture of a single step Galaxy evolution function is clearly quite crude. It does give, however, at least some indication of the overall trend of ψe^{ν} over time. Moreover, we improve the "resolution" by including more chronometers. The constraints we get, by complete analogy with the two chronometer case, are

$$\psi e^{\nu} = r(i,1) \quad \text{for} \quad t_{i-1} \leq t \leq t_i \tag{6.1}$$

where i runs from 1 to m, the total number of chronometers, t_i is defined by

$$t_i = \alpha_1(\tau_1 - \tau_{i+1}), \tag{6.2}$$

and $\alpha_1 = T/\tau_1$, as before. The boundary condition on t_i is $t_m=T$, which translates to $\tau_{m+1}=0$.

ii) Constraints on t_ν/T

From the above constraints we calculate the normalized effective nucleosynthesis rate over $t_{i-1} \leq t \leq t_i$ (see equation (3.1)):

$$\phi_i = \frac{r(i,1)\tau_1}{T\sum_{j=1}^{m} r(j,1)(\tau_j - \tau_{j+1})} \tag{6.3}$$

and the ratio of the mean time for formation of the elements t_ν (see equation (3.4)) to T

$$\frac{t_\nu}{T} = \frac{1}{2}\frac{\sum_{i=1}^{m} r(i,1)[(\tau_1 - \tau_{i+1})^2 - (\tau_1 - \tau_i)^2]}{\tau_1\sum_{j=1}^{m} r(j,1)(\tau_j - \tau_{j+1})}. \tag{6.4}$$

In the case of steady synthesis, r(i,1)=1 and, hence, $\sum_{i=1}^{m}\phi_i = 1/T$ and $t_\nu/T=1/2$, as expected.

Use of the upper limits on r(i,1) from equation (5.7) gives an upper limit on t_ν/T. We note the self-consistency of the approach in this case since these maximum r(i,1) are all calculated by averaging over the same number of half-lifes while t_ν/T is also computed by looking over the same number (α_1) of half-lifes for each nuclide. Furthermore, we expect α_1 to be greater than 1.0 since we will take $\tau_1 = \tau$ of $^{232}Th = 14.1$Gyr and T may be as large as 10Gyr (Meyer and Schramm 1985) or larger; thus, it makes sense to average over the same number of half-lifes for all nuclides, that is, it makes sense to use equation (5.7) to compute the maximum r(i,1).

Where it does not make sense to average over the same number of half-lifes is the case for the lower limit on t_ν/T. T may be as small as 1.8 Gyr (Meyer and Schramm 1985); therefore, α_1=0.13 if $\tau_1 = \tau$ of ^{232}Th. In this case we would only be averaging ψe^ν over 0.13 half-lifes of the various nuclides. This would not be appropriate for a nuclide like ^{244}Pu for which τ=0.08Gyr and which, consequently, has 22.5 half-lifes in 1.8Gyr. Since this would give an overestimate for $<\psi>_{\bar{\tau}_i,1}/<\psi>_{\bar{\tau}_1,1}$ and we want absolute lower limits on the r(i,1) to get a lower limit on t_ν/T, we must either restrict our set of chronometers to ones for which $\tau_1 < T$ or find an alternative means of getting a lower bound on t_ν/T. Since the lower bound on T of 1.8Gyr limits us to the two chronometers ^{244}Pu and ^{235}U (τ_{235}=0.70Gyr), and since there are fairly large uncertainities in the data for ^{244}Pu, we find it advisable to seek an alternative method for determining t_ν/T which can use the other, less uncertain, chronometers.

Such an alternative method is to consider two chronometers in the single step function of section Vi. We relax the constraint that the time intervals over which we integrate this model are $0 \leq t \leq t_1$ and $t_1 \leq t \leq T$. Instead we replace t_1 by aT, where $0 \leq a \leq 1$. Normalization yields

$$\phi(t) = \frac{1}{T[a + r(2,1)(1-a)]} \qquad \text{for } 0 \leq t \leq aT \tag{6.5}$$

and

$$\phi(t) = \frac{r(2,1)}{T[a + r(2,1)(1-a)]} \qquad \text{for } aT < t \leq T. \tag{6.6}$$

This $\phi(t)$ yields a mean age

$$\frac{t_\nu}{T} = \frac{1}{2}\frac{[a^2 + r(2,1)(1-a^2)]}{[a + r(2,1)(1-a)]}. \tag{6.7}$$

Let us now find extrema of t_ν/T at

$$a = \frac{r(2,1) - \sqrt{r(2,1)}}{r(2,1) - 1}, \tag{6.8}$$

since $0 \leq a \leq 1$. Substitution of equation (6.8) back into equation (6.7) yields

$$\frac{t_\nu}{T} = \frac{r(2,1) - \sqrt{r(2,1}}{r(2,1) - 1} \tag{6.9}$$

We can thus identify a with t_ν/T. The smallest possible value for r(2,1) (calculated from equation (5.9)) then gives a lower limit on t_ν/T. We will compare the results from equations (6.4) and (6.9). We may also use equation (6.9) to obtain upper limits on t_ν/T if we take maximum values for r(2,1).

iii) Explicit Constraints on T

From the Galaxy evolution constraints developed in the last two subsections, we are also able to solve equation (2.3) directly. Using equation (6.1), we find for the chronometric pair i and j

$$\frac{N_i(T + \Delta)}{N_j(T + \Delta)} = \frac{P_i}{P_j} e^{-(\lambda_i - \lambda_j)\Delta} e^{-(\lambda_i - \lambda_j)T} \frac{\lambda_j}{\lambda_i} \frac{\sum_{k=1}^{m} r(k,1)(e^{\lambda_i t_k} - e^{\lambda_i t_{k-1}})}{\sum_{n=1}^{m} r(n,1)(e^{\lambda_j t_n} - e^{\lambda_j t_{n-1}})}. \tag{6.10}$$

If we use equation (6.2) and assume $\alpha_1 = T/\tau_1$, we obtain

$$\frac{\sum_{k=1}^{m} r(k,1)\left(e^{-\lambda_i(\frac{\tau_{k+1}}{\tau_1})T} - e^{-\lambda_i(\frac{\tau_k}{\tau_1})T}\right)}{\sum_{n=1}^{m} r(n,1)\left(e^{-\lambda_j(\frac{\tau_{n+1}}{\tau_1})T} - e^{-\lambda_j(\frac{\tau_n}{\tau_1})T}\right)} = \frac{\lambda_i}{\lambda_j} \frac{e^{(\lambda_i - \lambda_j)\Delta}}{R(i,j)}. \tag{6.11}$$

This can be solved numerically for T. Use of maximum values for the r(k,1)'s gives an upper limit on T. We may alternatively take $t_1 = aT$ and $t_2 = T$ for the single step constraints of equations (6.5) and (6.6). These yield

$$\frac{(e^{-(1-a)\lambda_i T} - e^{-\lambda_i T}) + r(2,1)(1 - e^{-(1-a)\lambda_i T})}{(e^{-(1-a)\lambda_j T} - e^{-\lambda_j T}) + r(2,1)(1 - e^{-(1-a)\lambda_j T})} = \frac{\lambda_i}{\lambda_j}\frac{e^{(\lambda_i - \lambda_j)\Delta}}{R(i,j)}. \tag{6.12}$$

We can solve this numerically for limits on T.

VII. Data

The accuracy of any constraints in nucleocosmochronology rests fundamentally on the accuracy of the data. In Table 1, we present the best current estimates of decay rates, the ratios R(i,j), and resulting Δ^{max} 's for the Re/Os, Th/U, U/U, and Pu/U pairs.

i) Re/Os

The long-lived chronometric pair ^{187}Re/^{187}Os, first proposed for study by Clayton (1964), is unique because ^{187}Os is stable and has no direct contribution from the r-process since it is shielded from β-decay from below by ^{187}Re. Clearly, then, the formulae required to derive R(187,187) (see equation (3.10)) are different from those of other chronometric pairs.

R(187,187) is given by (Schramm 1974)

$$R(187,187) = 1 + \frac{(^{187}\mathrm{Os})_c}{^{187}\mathrm{Re}}, \tag{7.1}$$

where $(^{187}\mathrm{Os})_c$ is the cosmoradiogenic contribution to the ^{187}Os abundance, that is, the contribution arising from β-decay from ^{187}Re. The rest of the ^{187}Os comes from the s-process; hence, we denote this contribution $(^{187}\mathrm{Os})_s$. Since the ^{186}Os is not shielded and, consequently, not changing after nucleosynthesis, we can write

$$\frac{(^{187}\mathrm{Os})_c}{^{186}\mathrm{Os}} = \frac{^{187}\mathrm{Os}}{^{186}\mathrm{Os}} - \frac{(^{187}\mathrm{Os})_s}{^{186}\mathrm{Os}}. \tag{7.2}$$

By use of the so-called "local-approximation" in the s-process

$$(^{187}\mathrm{Os})_s\,(\sigma_{187})_{30\mathrm{keV}} \approx (^{186}\mathrm{Os})_s\,(\sigma_{186})_{30\mathrm{keV}}, \tag{7.3}$$

we may write

$$\frac{(^{187}\mathrm{Os})_c}{^{186}\mathrm{Os}} = \frac{^{187}\mathrm{Os}}{^{186}\mathrm{Os}} - \mathrm{f}_s, \tag{7.4}$$

after Yokoi et al. (1983), where

$$\mathrm{f}_s = \left(\frac{\sigma_{186}}{\sigma_{187}}\right)_{\mathrm{lab}} \mathrm{x}\ \mathrm{f}_\sigma\ \mathrm{x}\ \mathrm{f}_\mathrm{b}. \tag{7.5}$$

The σ's are neutron capture cross sections measured in the lab for ^{186}Os and ^{187}Os, f_σ is a factor allowing for conversion from lab cross sections to cross sections thermalized to a temperature of 30 keV with the nuclei starting at that level of excitation, that is, at conditions appropriate for the s-process inside stars, and f_b is a factor allowing for s-process branching in the W-Os region. Normalization of equation (7.4) by ^{186}Os/^{187}Re then gives the required ratio $(^{187}\mathrm{Os})_c/^{187}\mathrm{Re}$.

A major uncertainty in the Re/Os pair that limits its use is that astration greatly enhances the ^{187}Re β-decay rate over the lab rate by bound state decay (Takahashi and Yokoi 1982; Yokoi et al. 1983). To compute the Galaxy's age accurately from this pair thus requires a detailed Galaxy chemical evolution model to determine the amount of time ^{187}Re spends at various temperatures inside stars as well as an accurate estimate of λ_{187} as a function of temperature. Others have followed this tack (Yokoi et al. 1983). We will instead notice that the effect of astration is to increase λ_{187}, so that use of the lab λ_{187} in conjunction with equations (3.10) and (7.1) provides an upper limit on $\Delta^{max}_{187,187}$.

For the Re/Os pair, $\lambda_i = \lambda_{187}$ and $\lambda_j = 0$; hence, equation (4.16) becomes

$$\epsilon \lesssim \frac{1}{8}(1 - \frac{t_\nu}{T})^{-2}(\ln R(187,187))(1 + \epsilon)^2 + \frac{5}{312}(1 - \frac{t_\nu}{T})^{-3}(\ln R(187,187))^2(1 + \epsilon)^3 +$$

$$+ \frac{1}{288}(1 - \frac{t_\nu}{T})^{-4}(\ln R(187,187))^3(1 + \epsilon)^4 + \cdots \quad (7.6)$$

where we have made use of the fact that $\Delta^{max} - \Delta \approx \Delta^{max}$ (since $\Delta \lesssim$ 0.2Gyr (Symbalisty and Schramm 1981)) and of equation (3.10).

The significance of equation (7.6) lies in the fact that when we calculate the higher moment terms for Re/Os, *these terms will be independent of* λ_{187}. This puts us in the unique position of knowing the maximum possible corrections to $T - t_\nu$, as derived from $\Delta^{max}_{187,187} - \Delta$, better than we know $\Delta^{max}_{187,187} - \Delta$ itself because of the uncertainty in the effective λ_{187}. Furthermore, if we can show that the higher moment terms are all small, then the ^{187}Re/^{187}Os pair behaves as if it is long-lived, *independent of* λ_{187}.

The remaining uncertainties in the ^{187}Re/^{187}Os pair are the uncertainty in the amount of s-process branching in the W-Os region (f_b) (Arnould 1974; Arnould et al. 1984) and the uncertainty in the factor f_σ (Fowler 1973; Holmes et al. 1976; Woosley and Fowler 1979) in equation (7.5). Yokoi et al. (1983) estimate that the combined uncertainties in f_σ and f_b yield a range of $0.41 \lesssim f_s \lesssim 0.58$. Arnould et al. (1984) argue, however, that s-process branching uncertainties lead to much larger ranges $0.18 \lesssim f_s \lesssim 0.69$ for $1 \lesssim f_\sigma \lesssim 1.15$ or $0.15 \lesssim f_s \lesssim 0.69$ for $0.81 \lesssim f_\sigma \lesssim 0.83$. We could thus take an extreme range $0.15 \lesssim f_s \lesssim 0.69$. In what follows, we will prefer $0.41 \lesssim f_s \lesssim 0.58$ but will also consider the effect of the $0.15 \lesssim f_s \lesssim 0.69$ range.

The other information we need to calculate R(187,187) are the meteoritic ratios $({}^{187}Os/{}^{186}Os)_{T+\Delta}$ and $({}^{186}Os/{}^{187}Re)_{T+\Delta}$. A fairly recent, self-consistent determination of $({}^{187}Os/{}^{186}Os)_{T+\Delta}$ and $({}^{187}Re/{}^{186}Os)_{T+\Delta}$ is that of Luck et al. (1980), who obtain 0.805±0.011 and 3.2(±10%) for these two numbers, respectively. With these numbers, we find $1.06 \lesssim R(187,187) \lesssim 1.14$ for the range $0.41 \lesssim f_s \lesssim 0.58$. For the extreme range $0.15 \lesssim f_s \lesssim 0.69$ we find $1.03 \lesssim R(187,187) \lesssim 1.23$

ii) Th/U

The ${}^{232}Th/{}^{238}U$ production ratio is quite uncertain. Symbalisty and Schramm (1981) give the wide range $1.9^{+0.2}_{-0.4}$. Thielemann et al. (1983a) include β-delayed fission in their calculations of P_{232}/P_{238} and find a value of 1.4. Meyer et al. (1985) find in a self-consistent calculation less β-delayed fission than Thielemann et al. which suggests a higher production ratio; thus, we will take 1.4 as a lower limit on the production ratio. Because the new value for the sum of the α-decay and β-decay branching ratio for ${}^{250}Cm$ (a progenitor of ${}^{238}U$) is 35% (Schmorak 1981), up from the value of 10% used by Seeger and Schramm, we lower the Seeger and Schramm value from 1.9 to 1.8. We then use this 1.8 as an upper limit on P_{232}/P_{238} since the original calculations of Seeger and Schramm, included no β-delayed fission and some β-delayed fission undoubtedly occurred. We will also take 1.6 as a compromise best value. We note, however, that the calculation of Meyer et al. did not include fission barrier penetration so that a lower best value, nearer the value of Thielemann et al., should perhaps be favored.

SW argue that the present solar system value for ${}^{232}Th/{}^{238}U$ is 3.9. They note that terrestrial lead isotopic ratios evolved from a ${}^{232}Th/{}^{238}U$ ratio in the range 3.7

to 4.1 with 3.9 as the best value. Since this ratio seems to apply to essentially all rocks of all ages, they conclude that it is a ratio characteristic of the earth as a whole and is the one consistent with the standard 4.55 Gyr age of the solar system. They also note that the lead in meteorites such as Nuevo Laredo evolved from a $^{232}Th/^{238}U$ ratio identical to the terrestrial lead ratio. C1 chondrites, however, give perhaps the most primative solar system abundances not obtainable from observations of the solar atmosphere (Anders 1971). We have data on thorium abundances in the C1 chondrites Orgueil, Ivuna, Alais, and Tonk (Tatsumoto et al. 1976; Morgan and Lovering 1968). From this set we neglect those values of Morgan and Lovering which are anomalously high since they probably resulted from contamination (Anders 1986). We also have data from the same set of meteorites on the uranium abundances (Tatsumoto et al. 1976; Krähenbühl et al. 1973; Morgan and Lovering 1968; Reed and Allen 1966; Reed, Kogoshi, and Turkevich 1960). We have again neglected those values of Morgan and Lovering which are anomalous. Individual meteorite (atomic) ratios we find from the Morgan and Lovering and Tatsumoto et al. data are 4.10 (Alais); 2.36, 3.80, and 3.69 (Ivuna); 3.49 and 4.41 (Orgueil); and 2.98 (Tonk). The wide spread shows that chemical fractionation probably played a large role in the Th/U ratios in C1 chondrites. From averages of the two sets of data from all five sources (nine thorium measurements and 16 uranium measurements), we find an abundance ratio of 3.72. This value is contained within the SW lower limit; hence, we choose 3.7 as our lower limit on the Th/U ratio. We also choose 4.1 as an upper limit since it contains the 3.9 value from leads and the values from five of the six individual meteorite measurements. Since the Th/U ratio is not an isotopic ratio, simply dividing Th abundances in C1 chondrites by corresponding U abundances may not take into account differences due to the different chemical natures of the two elements, as possibly evidenced by the variations seen in the individual C1 chondrite measurements. The ratio obtained from terrestrial leads, however, integrates over variations found in the Th/U ratio measured directly in rocks (SW). As a consequence, we still take 3.9 and believe it is the best and most consistent value for cosmochronological purposes. Our present day $^{232}Th/^{238}U$ ratio is thus 3.9 ± 0.2, the SW range.

iii) U/U

The value given by Symbalisty and Schramm (1981) for the production ratio of $^{235}U/^{238}U$ is $1.5^{+0.6}_{-0.6}$. SW give the range $1.5^{+0.4}_{-0.3}$. Thielemann et al. (1983a) get 1.24, which is easily contained in both of the above ranges. The calculation of Meyer et al. (1985), as mentioned above, gives less delayed fission than the calculation of Thielemann et al., which indicates a production ratio closer to 1.5 and a lower limit given by the Thielemann et al. value; thus, the Symbalisty and Schramm lower limit is probably too low. Furthermore, since the effect of delayed fission is probably to decrease P_{232}/P_{238} from roughly 1.5, we expect the Symbalisty and Schramm upper limit to be too high. We thus choose the SW range as the best range for this production ratio.

The terrestrial abundance ratio for the $^{235}U/^{238}U$ pair is quite well known. The standard value from uranium-bearing ores is 1/137.88. Cowan and Adler (1976) in their precision mass spectrometry of a large number of ore samples from around the world found an approximately bimodal distribution of $^{235}U/^{238}U$ around the standard value and around roughly 1/137.95. The latter value was derived mainly from ores from the Colorado Plateau, which may have been subject to chemical differentiation of the U isotopes, although Cowan and Adler found at the time of their paper that insufficient data existed to make that conclusion. In any case, the results we may derive from Cowan and Adler's ^{235}U weight percentages, $^{235}U/^{238}U = 1/(137.88^{+0.09}_{-0.08})$, are contained in the value $1/(137.88\pm0.14)$, given in Barnes et al. (1972), which includes lunar data.

Arden (1977) found large variations, up to about 29%, in $^{235}U/^{238}U$ in a variety of chondritic meteorites (including Allende). Tatsumoto et al. (1980) and Tatsumoto and Shimamura (1980) found approximately 7% variations in $^{235}U/^{238}U$ in the Allende meteorite, which they interpreted as evidence for the presence of live ^{247}Cm at $T + \Delta$ which then fractionated to varying degrees from the U to give the variations observed. This ^{247}Cm may have been injected into the solar system material in the same event that contributed the live ^{26}Al, ^{107}Pd, and ^{129}I. Chen and Wasserburg (1981), however, find that $^{235}U/^{238}U$ ratios from Allende inclusions agree quite well with the standard value. From these results, they conclude that live ^{247}Cm may indeed have been present at $T + \Delta$, but only at low levels so that Cm/U fractionation would have had a negligible effect on the U/U ratio. The agreement between the Chen and Wasserburg values and the standard value and the possibility of isotopic contamination in the analysis of Tatsumoto et al. (Symbalisty and Schramm 1981) lead us to conclude that the present day $^{235}U/^{238}U$ solar system abundance ratio is $1/(137.88\pm0.14)$, the concordant terrestrial-lunar value.

iv) Pu/U

Symbalisty and Schramm give $0.9^{+0.1}_{-0.2}$ for the ^{244}Pu/^{238}U production ratio. This chronometric pair is extremely sensitive to delayed fission, however, as evidenced by Thielemann et al.'s value of 0.12. The results of Meyer et al. tend to indicate a value somewhere in between these two ranges. We will accept for the present a large uncertainty in the ^{244}Pu/^{238}U production ratio and, thus, take the range $0.12 \lesssim P_{244}/P_{238} \lesssim 1.0$, with 0.56 as a compromise best value. Again, the fact that barrier penetration was not included in Meyer et al. might lead us to favor a lower best value.

The ^{244}Pu/^{238}U abundance ratio is found from decay products of ^{244}Pu. Ganapathy and Grossman (1976) proposed that the solar system ^{244}Pu/^{238}U abundance ratio be determined from coarse-grained Ca-Al-rich inclusions from Allende since they found that in ten such coarse-grained inclusions the average relative abundance of 21 refractory elements was unfractionated with respect to C1 chrondrite abundances. The advantage of these inclusions is that absolute concentrations of these elements are some 18 times greater than the C1 abundances. Drozd et al. (1977) obtained a ratio of 0.016 at $T + \Delta$ from a coarse-grained inclusion from Allende, which agrees well with 0.015 at $T + \Delta$ obtained by Podosek (1972) from St. Severin. Marti et al. (1977) found a Pu/Nd atomic ratio of 8.2×10^{-5} from a coarse-grained inclusion in Allende. Burnett et al. (1982) use this number to find 0.0040 for Pu/U at $T + \Delta$. Boynton (1978) argued that the apparently high Th/U ratios obtained from coarse-grained inclusions probably resulted from U depletion. This suggests that U abundances used by Drozd et al.

(Shirck 1975) may be systemically too low and, thus, that we should prefer 0.004 to 0.016 for the Pu/U ratio at $T + \Delta$. This seems to be confirmed by the results of Hudson (1981) who got 0.005 from a re-analysis of St. Severin. This value is also corroborated by Marti et al. (1977) who proposed from analyses of achrondritic meteorites a value of 0.004. We thus choose the value suggested by the work of Hudson et al. (1982) of 0.005 ± 0.001. We note, however, that there is potential for much greater uncertainty than is included in this range.

The decay rates we use throughout this work are $\lambda_{244} = 8.47\pm0.27 \times 10^{-10}\,\mathrm{yr}^{-1}$ (Fields et al. 1966), $\lambda_{235} = 9.8485\pm0.0135 \times 10^{-10}\,\mathrm{yr}^{-1}$ (Jaffey et al. 1971), $\lambda_{238} = 1.5512\pm0.0017 \times 10^{-10}\,\mathrm{yr}^{-1}$ (Jaffey et al. 1971), and $\lambda_{187} = 1.59^{+0.05}_{-0.04} \times 10^{-10}\,\mathrm{yr}^{-1}$ (Linder et al. 1986). The errors on λ_{235} and λ_{238} are two times those quoted in Jaffey et al. since these authors claim systematic errors should no more than double their quoted errors. The quantity t_{88} has been well established as being between 4.5 Gyr and 4.6 Gyr since the lead-lead age determination of Patterson (1955). The best value now is the standard 4.55 Gyr (see for example Tatsumoto et al. 1976).

VIII. Results

i) Ratios of Average Nucleosynthesis Rates

Table 2 shows the results for $<\psi>_{\bar{\tau}_i,I}/<\psi>_{\bar{\tau}_j,J}$ as calculated from equation (5.7) for Δ=0.0Gyr, 0.1Gyr, and 0.2Gyr. Table 3 shows the results for $<\psi>_{0,I}/<\psi>_{0,J}$ as calculated from equation (5.9). The numbers in Table 3 can be directly compared with those in Table 1 of Reeves and Johns (1976).

The numbers in Table 2 are the maximum values for the ratios and are all relatively close to 1.0, which suggests that ψe^{ν} was roughly steady (to within a factor of two or so) over most of T. Errors allow these numbers to vary by a factor of 3, except for the Pu/U ratio, which is extremely sensitive to Δ and to uncertainties in its input data and, consequently, varies from 1.0 by as much as a factor of 16.

The numbers in Table 3 for T=2.2 Gyr are the minimum values for the ratios. Except for Pu/U, these are also within a factor of two or three of 1.0, which again suggests ψe^{ν} was roughly steady over most of T. The other numbers in Table 2, for T=5.0Gyr, 10.0Gyr, and 15.0Gyr, show even better agreement with the idea of rough constancy of ψe^{ν}. We thus conclude that ψe^{ν} was constant over T to within a factor of two or three, except perhaps for the last events which produced the ^{244}Pu that we infer was present at t=T. We note, however, that the large range for the Pu/U ratio comes from a large range in the ^{244}Pu input data; hence, we should expect much better numbers when the Pu/U production ratio is better determined.

ii) t_ν/T

From Table 2, we can assign maximum values to the numbers r(i,1) defined in section V. We take for i=1 ^{232}Th, for i=2 ^{238}U, for i=3 ^{235}U, and i=4 for ^{244}Pu. Allowing for the extreme values, we find r(1,1)=1.0, r(2,1)=2.49, r(3,1)=4.96, and r(4,1)=40.5. From equation (6.4) then we get an absolute maximum $t_\nu/T = 0.68$. If we do not include r(4,1) because of the uncertainties in the ^{244}Pu data, we find a maximum $t_\nu/T = 0.63$. If we use equation (6.9), the t_ν/T from the single step function, and r(2,1)=2.49, we find $t_\nu/T = 0.61$. Finally, if we again use equation (6.9), but this time for the U/U pair, we find $t_\nu/T = 0.59$. Since the U/U time scales are the optimal ones for exploring overall Galaxy evolution, we prefer 0.59 as the upper limit on t_ν/T.

From Table 3 for T=1.6 Gyr, we get the following minimum values for the r(i,1)'s: r(1,1)=1.0, r(2,1)=0.59, r(3,1)=0.22, and r(4,1)=0.035. To use equation (6.4), we take r(3,3)=1.0 and r(4,3)=0.16, that is, our restricted set of chronometers. These numbers give us $t_\nu/T = 0.45$. From equation (6.9) and r(2,1)=0.59, we find $t_\nu/T = 0.43$. From equation (6.9) and use of r(3,2) in place of r(2,1), we find $t_\nu/T = 0.38$. The agreement of the first two of these limits leads us to favor a lower t_ν/T of 0.43.

To summarize, then, our range is $0.43 \lesssim t_\nu/T \lesssim 0.59$.

iii) ϵ and T

The lower limit on $(\Delta^{max}_{238,232} - \Delta)$ is 2.2 Gyr. From the lower limit on t_ν/T of 0.43 and $\Delta \lesssim 0.2$ Gyr, we find $T \gtrsim 3.9$Gyr or $T_{Gal} \gtrsim 8.7$Gyr. When we use equation (6.12), r(2,1)=0.56, and a=0.43, the limit on T_{Gal} becomes $T_{Gal} \gtrsim 8.9$ Gyr. Using equation (6.12), r(3,2)=0.30, and a=0.43, we find $T_{Gal} \gtrsim 8.4$Gyr. Higher moment terms thus increase the lower limit on T_{Gal} by only 2% or so from the single event age corrected for $t_\nu/T = 0.43$.

Tables 4 and 5 show values of ϵ, T, and T_{Gal}, as calculated from equations (7.6) and (4.17), for values of t_ν/T from 0.2 to 0.8. The values in Table 4 come from an R(187,187)=1.14. The values in Table 5 come from an R(187,187)=1.23, which is the extreme value allowed by the Arnould et al. (1984)

value of f_s. From Table 4, we find $\epsilon \lesssim 0.13$ for $t_\nu/T = 0.59$, which indicates that the Re/Os pair is long-lived to better than 13%. We also find for $t_\nu/T = 0.59$ that $T \lesssim 23.3$ Gyr. This yields $T_{Gal} \lesssim 28.1$Gyr. From Table 5, we find $T_{Gal} \lesssim 46.3$ Gyr. An upper limit of 46.3, or even of 28.1 Gyr, is not particularly useful. We do note, however, that for $t_\nu/T = 0.50$, we get $T_{Gal} \lesssim 23.0$Gyr for R(187,187)=1.14. An upper limit of 23.0 Gyr would be a fairly useful number since it is near the limits imposed by Thielemann et al. (1983b).

Because of the uncertainties in R(187,187), we may ask whether the Th/U pair might give us a better upper limit. Using an upper limit on $\Delta^{max}_{232,238}$ of 5.5 Gyr, we find no convergence of equation (4.16) to fourth order in ϵ. We thus conclude that Th/U cannot be considered a long-lived pair since ϵ is greater than or comparable to 1 for the upper limit on $\Delta^{max}_{232,238}$. As a consequence, *any upper limit on* T_{Gal} *derived from Th/U will be model-dependent.*

From equation (6.11), R(238,232)=1.79, and the upper limits on the r(i,1)'s (but not including r(4,1) because of the uncertainties in the Pu/U pair), we find $T_{Gal} \lesssim 36.2$Gyr. From equation (6.12), r(2,1)=2.49, and a=0.59, we find $T_{Gal} \lesssim 23.6$ Gyr, which lowers the Re/Os upper limit of 27.1 Gyr. We note that using $\Delta^{max}_{232,238} = 5.5$ Gyr gives $T_{Gal} = 18.0$ Gyr in the single event model plus correction for $t_\nu/T = 0.59$. Higher moment terms thus increase the Th/U upper limit on T by 42% from 13.2 Gyr to 18.8 Gyr. Clearly the Th/U pair is not long-lived in this case.

To summarize, we find a model-independent range $8.7 \text{ Gyr} \lesssim T_{Gal} \lesssim 28.1$Gyr. Again we must emphasize that uncertainties in R(187,187) do not allow us to claim this upper limit as absolutely firm, despite its large value. The more model-dependent equations of section VIiii narrow this range somewhat to $8.9\ \mathrm{Gyr} \lesssim T_{Gal} \lesssim 23.6\ \mathrm{Gyr}$.

IX. Conclusions

Qualitative conclusions we may derive from this work are that 1) upper and lower limits on the age of the Galaxy can be obtained that depend only on t_ν/T but are independent of any other model-dependent information and of the major uncertainties in cosmochronological input data, 2) cosmochronological input data itself can provide the limits on t_ν/T necessary to obtain the limits on the age of the Galaxy via a method which depends only on the lifetimes of the chronometers in question, not on any biases about the form ψe^ν should take, and 3) the cosmochronology input data can also give rough constraints on the overall constancy of ψe^ν over T.

Quantitatively, we find limits for t_ν/T of $0.43 \lesssim t_\nu/T \lesssim 0.59$. From these numbers we find that we can place a lower limit on T_{Gal} of 8.7 Gyr and an upper limit of 28.1 Gyr. The uncertainty in R(187,187) does not allow us to claim this upper limit as absolutely firm, however. We also find quantitatively that ψe^ν probably did not vary by more than a factor of three over most of T but that uncertainties in the short-lived chronometers do not preclude the possibility of large changes in ψe^ν for t near T.

The range for T_{Gal} that we find is discouragingly large. This really reflects, however, the uncertainties in the cosmochronological data rather than any real failing of the methods described in this paper. In particular, we note that calculation of $<\psi>_{\bar{\tau}_i,i}/<\psi>_{\bar{\tau}_j,j}$ depends on R(i,j), not on ln(R(i,j)) as does the calculation of $\Delta_{i,j}^{max}$; thus, parameters giving constraints on Galaxy chemical evolution are quite sensitive to errors in the production and abundance ratios. The upper limits we obtain are also sensitive to the extremely uncertain parameter R(187,187). We can expect better limits when these input data errors are narrowed. Indeed, we have noted that if input data improve to give $t_\nu/T \approx 0.50$, as expected from the work of Hainebach and Schramm (1977), we find a useful upper limit on T_{Gal} of about 23.0 Gyr.

For comparison with our model-independent range, we may consider some model-dependent age ranges. In particular, we can determine ranges on T_{Gal} for our range on $\Delta_{232,238}^{max}$ of 1.8 to 5.5 Gyr by looking at Hainebach and Schramm's figures 2, 4, and 5 which represent their solutions to the standard Galaxy evolution model (Tinsley 1975), the metal-enhanced star formation Galaxy evolution model (Talbot and Arnett 1973), and the halo-disk Galaxy evolution model (Ostriker and Thuan 1975), respectively. For the standard model we find a range for T_{Gal} of 8.7 to 18.8 Gyr. For the metal-enhanced model we find the range 9.0 to 18.5 Gyr. Finally, for the halo-disk model we find the range 8.7 to 18.5 Gyr. These ranges are certainly narrower than our range; however, they are dependent upon the models and on the parameters chosen for those models. Our range is independent of such considerations.

We should note in closing that an absolutely accurate upper limit is probably not attainable from this method since use of equation (3.10) and the lab ^{187}Re decay rate can only give an upper limit on $\Delta^{max}_{187,187}$. Once we obtain the best possible data for the Re/Os pair, we might be able to improve our upper limit on T_{Gal} through detailed Galaxy evolution models which can determine the amount of astration of ^{187}Re and, hence, the true value for the effective ^{187}Re decay rate. Alternatively, we may use the Th/U pair. As we have shown, however, this pair cannot be considered long-lived in the calculation of upper limits for T_{Gal}; hence, any upper limit we derive from it will also be model-dependent. It will be a sign of great progress, however, when our major concern is the model-dependency of our upper limits rather than uncertainties in the data.

Acknowledgements

We would like to acknowledge useful discussions with K. Takahashi, F.-K. Thielemann, J.W. Truran, and G.J. Wasserburg. We are especially grateful to E. Anders and L. Grossman for help in tracking down some of the meteorite data. This work was supported in part by NSF grant AST 83-13128 at the University of Chicago. One of us (B.S.M.) would like to acknowledge the support of a National Science Foundation Graduate Fellowship.

Appendix

In order to find maximum values for μ_n/T^n when n is odd, we note that the factor $(t - t_\nu)^n$ in the integral in equation (3.8) is odd about $t=t_\nu$. To get the largest μ_n possible, then, we want the contribution from $t<t_\nu$ to be as small as possible and the contribution from $t\geq t_\nu$ to be as large as possible. We have found in section VIIIi that ψe^ν, and hence $\phi(t)$, probably did not vary by more than a factor of three (except possibly over a very small time scale just before t=T). Since $\phi(t)=1/T$ for a steady synthesis solution (see equations (3.1) and (3.2)), we can safely assume that $\phi(t) \gtrsim 1/3T$ for $t<t_\nu$ and $\phi(t) \lesssim 3/T$ for $t\geq t_\nu$. From these numbers, we find

$$\frac{\mu_3}{T^3} \lesssim \frac{3}{4}\left(1 - \frac{t_\nu}{T}\right)^4 - \frac{1}{12}\left(\frac{t_\nu}{T}\right)^4 . \tag{A.1}$$

With the lower bound on t_ν/T of 0.43 from section VIIIii, we find

$$\frac{\mu_3}{T^3} \lesssim \frac{1}{13.1} . \tag{A.2}$$

Table 1

Pair	$\lambda_i(\mathrm{Gyr}^{-1})$	$\lambda_j(\mathrm{Gyr}^{-1})$	R(i,j)	$\Delta_{i,j}^{max}(\mathrm{Gyr})$
^{187}Re/^{187}Os	0.0162(+0.0003,-0.0003)	-----	1.03-1.23	1.8-13.4
^{232}Th/^{238}U	0.0495(+0.0000,-0.0000)	0.1551(+0.0002,-0.0002)	0.67(+0.11,-0.11)	3.8(+1.7,-1.4)
^{235}U/^{238}U	0.985(+0.009,-0.009)	0.1551(+0.0002,-0.0002)	4.7(+1.3,-0.9)	1.9(+0.3,-0.3)
^{244}Pu/^{238}U	8.47(+0.27,-0.27)	0.1551(+0.0002,-0.0002)	112(+138,-92)	0.57(+0.12,-0.21)

Table 2. Maximum $<\psi>_{\bar{\tau}_i}/<\psi>_{\bar{\tau}_j}$'s

Pair	Δ=0.0 Gyr	Δ=0.1 Gyr	Δ=0.2 Gyr
U/Th	2.09(+0.35,-0.46)	2.11(+0.36,-0.46)	2.13(+0.36,-0.47)
U/U	1.36(+0.32,-0.29)	1.48(+0.35,-0.32)	1.61(+0.38,-0.35)
Pu/U	0.49(+2.41,-0.28)	1.12(+5.73,-0.65)	2.56(+13.7,-1.52)

Table 3. $<\psi>_{0,i}/<\psi>_{0j}$

Pair	Δ(Gyr)	T=2.2 Gyr	T=5.0 Gyr	T=10.0 Gyr	T=15.0 Gyr
U/Th	0.0	0.75(+0.13,-0.16)	0.85(+0.14,-0.19)	1.03(+0.18,-0.23)	1.21(+0.21,-0.27)
	0.1	0.75(+0.13,-0.16)	0.86(+0.15,-0.19)	1.05(+0.18,-0.23)	1.23(+0.21,-0.27)
	0.2	0.76(+0.13,-0.17)	0.87(+0.15,-0.19)	1.06(+0.18,-0.23)	1.24(+0.21,-0.27)
U/U	0.0	0.44(+0.10,-0.10)	0.74(+0.17,-0.16)	1.07(+0.25,-0.23)	1.23(+0.29,-0.27)
	0.1	0.48(+0.11,-0.10)	0.80(+0.19,-0.17)	1.16(+0.28,-0.25)	1.33(+0.32,-0.29)
	0.2	0.52(+0.12,-0.11)	0.87(+0.21,-0.19)	1.27(+0.30,-0.27)	1.45(+0.34,-0.31)
Pu/U	0.0	0.14(+0.68,-0.08)	0.26(+1.27,-0.15)	0.38(+1.87,-0.22)	0.44(+2.16,-0.25)
	0.1	0.32(+1.62,-0.19)	0.60(+3.04,-0.35)	0.88(+4.47,-0.51)	1.01(+5.14,-0.58)
	0.2	0.74(+3.87,-0.44)	1.38(+7.24,-0.82)	2.02(+10.6,-1.19)	2.31(+12.2,-1.37)

Table 4. Results for R(187,187) = 1.14[1]

$\frac{t_\nu}{T}$	ϵ	T(Gyr)	T_{Gal}(Gyr)
0.2000	0.0276	10.9	15.7
0.2100	0.0284	11.0	15.8
0.2200	0.0292	11.2	16.0
0.2300	0.0300	11.3	16.1
0.2400	0.0308	11.5	16.3
0.2500	0.0317	11.6	16.4
0.2600	0.0327	11.8	16.6
0.2700	0.0337	12.0	16.8
0.2800	0.0347	12.1	16.9
0.2900	0.0357	12.3	17.1
0.3000	0.0369	12.5	17.3
0.3100	0.0380	12.7	17.5
0.3200	0.0393	12.9	17.7
0.3300	0.0406	13.1	17.9
0.3400	0.0420	13.3	18.1
0.3500	0.0434	13.6	18.4
0.3600	0.0449	13.8	18.6
0.3700	0.0465	14.0	18.8
0.3800	0.0482	14.3	19.1
0.3900	0.0500	14.6	19.4
0.4000	0.0519	14.8	19.6
0.4100	0.0539	15.1	19.9
0.4200	0.0560	15.4	20.2
0.4300	0.0583	15.7	20.5
0.4400	0.0607	16.0	20.8
0.4500	0.0633	16.3	21.1
0.4600	0.0661	16.7	21.5
0.4700	0.0690	17.1	21.9
0.4800	0.0722	17.4	22.2
0.4900	0.0756	17.8	22.6
0.5000	0.0792	18.2	23.0
0.5100	0.0832	18.7	23.5
0.5200	0.0875	19.2	24.0
0.5300	0.0921	19.6	24.4
0.5400	0.0972	20.2	25.0
0.5500	0.1027	20.7	25.5
0.5600	0.1087	21.3	26.1
0.5700	0.1153	21.9	26.7
0.5800	0.1227	22.6	27.4
0.5900	0.1308	23.3	28.1
0.6000	0.1399	24.1	28.9
0.6100	0.1501	24.9	29.7
0.6200	0.1616	25.8	30.6
0.6300	0.1747	26.8	31.6
0.6400	0.1898	27.9	32.7
0.6500	0.2073	29.2	34.0
0.6600	0.2279	30.5	35.3
0.6700	0.2527	32.1	36.9
0.6800	0.2831	33.9	38.7
0.6900	0.3216	36.0	40.8
0.7000	0.3727	38.7	43.5
0.7100	0.4464	42.2	47.0
0.7200	0.5751	47.6	52.4
0.7300	******		
0.7400	******		
0.7500	******		
0.7600	******		
0.7700	******		
0.7800	******		
0.7900	******		
0.8000	******		

[1] Asterisks indicate that there is no convergence of equation (7.6) to fourth order in ϵ.

Table 5. Results for R(187,187) = 1.23[1]

$\frac{t_\nu}{T}$	ϵ	T(Gyr)	T_{Gal}(Gyr)
0.2000	0.0458	17.5	22.3
0.2100	0.0472	17.7	22.5
0.2200	0.0485	18.0	22.8
0.2300	0.0500	18.2	23.0
0.2400	0.0515	18.5	23.3
0.2500	0.0530	18.8	23.6
0.2600	0.0547	19.0	23.8
0.2700	0.0564	19.3	24.1
0.2800	0.0582	19.6	24.4
0.2900	0.0601	19.9	24.7
0.3000	0.0621	20.3	25.1
0.3100	0.0643	20.6	25.4
0.3200	0.0665	20.9	25.7
0.3300	0.0688	21.3	26.1
0.3400	0.0713	21.7	26.5
0.3500	0.0740	22.1	26.9
0.3600	0.0768	22.5	27.3
0.3700	0.0797	22.9	27.7
0.3800	0.0829	23.3	28.1
0.3900	0.0862	23.8	28.6
0.4000	0.0898	24.3	29.1
0.4100	0.0937	24.8	29.6
0.4200	0.0977	25.3	30.1
0.4300	0.1021	25.8	30.6
0.4400	0.1069	26.4	31.2
0.4500	0.1120	27.0	31.8
0.4600	0.1175	27.6	32.4
0.4700	0.1234	28.3	33.1
0.4800	0.1299	29.0	33.8
0.4900	0.1370	29.8	34.6
0.5000	0.1447	30.6	35.4
0.5100	0.1532	31.4	36.2
0.5200	0.1626	32.3	37.1
0.5300	0.1731	33.3	38.1
0.5400	0.1847	34.4	39.2
0.5500	0.1979	35.6	40.4
0.5600	0.2127	36.8	41.6
0.5700	0.2298	38.2	43.0
0.5800	0.2495	39.7	44.5
0.5900	0.2728	41.5	46.3
0.6000	0.3007	43.4	48.2
0.6100	0.3350	45.7	50.5
0.6200	0.3789	48.5	53.3
0.6300	0.4386	51.9	56.7
0.6400	0.5307	56.8	61.6
0.6500	0.7841	68.1	72.9
0.6600	******		
0.6700	******		
0.6800	******		
0.6900	******		
0.7000	******		
0.7100	******		
0.7200	******		
0.7300	******		
0.7400	******		
0.7500	******		
0.7600	******		
0.7700	******		
0.7800	******		
0.7900	******		
0.8000	******		

[1] Asterisks indicate there is no convergence in equation (7.6) to fourth order in ϵ.

References

Anders, E. 1971, *Geochim. Cosmochim. Acta,* **35** , 516.

Anders, E. 1986, private communication.

Anders, E., and Ebihara, M. 1982, *Geochim. Cosmochim. Acta,* **46** , 2363.

Arden, J. W. 1977, *Nature,* **269** , 788.

Arnould, M. 1974, *Astron. Ap.,* **31** , 371.

Arnould, M., Takahashi, K., and Yokoi, K. 1984, *Astr. Ap.,* **137** , 51.

Barnes et al. 1972, *Geochim. Cosmochim. Acta Suppl.,* **3** , 1465.

Boynton, W. V. 1978, *Earth Planet. Sci. Lett.,* **40** , 63.

Burbidge, E. M., Burbidge, G. R., Fowler, W. A., and Hoyle, F. 1957, *Rev. Mod. Phys.,* **29** , 547.

Burnett, D. S., Stapanian, M. I., and Jones, J. H. 1982, in *Essays in Nuclear Astrophysics,* eds. C. A. Barnes, D. D. Clayton, and D. N. Schramm (Cambridge: Cambridge University Press), p. 141.

Chen, J. H., and Wasserburg, G. J. 1981, *Lun. Planet. Sci.,* **XII** , Abstracts 132.

Clayton, D. D. 1964, *Ap. J.,* **139** , 637.

__________. 1984a, in *Nucleosynthesis,* ed. W. D. Arnett and J. W. Truran (Chicago: University of Chicago Press).

__________. 1984b, *Ap. J.,* **285** , 411.

__________. 1985, *Ap. J.,* **288** , 567.

Cowan, G. A., and Adler, H. H. 1976, *Geochim. Cosmochim. Acta,* **40** , 1487.

Drozd, R. J., Morgan, C. J., Podosek, F. A., Poupeau, G., Shirk, J. R., and Taylor, G. J. 1977, *Ap. J.,* **212** , 567.

Fields, P. R., Friedman, A. M., Milsted, J., Lerner, J., Stevens, C. M., and

Metta, D. 1966, *Nature,* **212** , 131.

Fowler, W. A. 1972, in *Cosmology, Fusion, and Other Matters,* ed. F. Reines (Boulder: Colorado Associated Universities Press).

__________. 1973, in *Explosive Nucleosynthesis,* ed. D. N. Schramm and W. D. Arnett (Austin: University of Texas Press), p.300.

Fowler, W. A., and Hoyle, F. 1960, *Ann. Phys.,* **10** , 280.

Ganapathy, R., and Grossman, L. 1976, *Earth Planet. Sci. Lett.,* **31** , 386.

Hainebach, K., and Schramm, D. N. 1977, *Ap. J.,* **212** , 347.

Hohenburg, C. M. 1969, *Science,* **156** , 233.

Holmes, J. A., Woosley, S. E., Fowler, W. A., and Zimmerman, B. A. 1976, *Atomic Data and Nuclear Data Tables,* **18** , 306.

Hudson, B. 1981, Ph. D. Thesis, Washington University.

Hudson, B., Hohenberg, C. M., Kennedy, B. M., and Podosek, F. A. 1982, *Lun. Planet. Sci.,* **XIII** , 346.

Jaffey, A. H., Flynn, D. F., Glendenin, L. E., Bentley, W. C., and Essling, A. M. 1971, *Phys. Rev.,* **C4** , 1889.

Kohman, T. P. 1972, *Proc. Cosmochemistry Symp.,* (Dordrecht: Reidel).

Krähenbühl, U., Morgan, J. W., Ganapathy, R., and Anders, E. 1973, *Geochim. Cosmochim. Acta,* **37** , 1353.

Lindner, M., Leich, D. A., Borg, R. J., Russ, G. P., Bazan, J. M., Simons, D. S., and Date, A. R. 1986, *Nature,* **320** , 246.

Luck, J. M., Birch, J. L., and Allegre, C. J. 1980, *Nature,* **283** , 256.

Marti, K., Lugmair, G. W., and Scheinin, N. B. 1977, in *Lunar Science,* **VIII,** 619.

Meyer, B. S., Howard, W. M., Mathews, G. J., Moller, P., and Takahashi, K. 1985, in *Recent Advances in the Study of Nuclei Away From Stability,* eds. R. Meyer and D. Brenner (Washington: American Chemical Society Press) in press.

Meyer, B. S., and Schramm, D. N. 1985, in *Nucleosynthesis and Its Implications on Nuclear and Particle Physics,* eds. J. Audouze and N. Mathieu (Dordrecht: Reidel) in press.

Morgan, J. W., and Lovering, J. F. 1968, *Talanta,* **15** , 1079.

Ostriker, J. P., and Thuan, T. X. 1975, *Ap. J.,* **202** , 353.

Patterson, C. C. 1955, *Geochim. Cosmochim. Acta,* **7** , 151.

Podosek, F. A. 1972, *Geochim. Cosmochim. Acta,* **36** , 755.

Reed, G. W., and Allen, R. O. 1966, *Geochim. Cosmochim. Acta,* **30** , 779.

Reed, G. W., Kogoshi, K., and Turkevich, A. 1960, *Geochim. Cosmochim. Acta,* **20** , 122.

Reeves, H., and Johns, O. 1976, *Ap. J.,* **206** , 958.

Rutherford, E. 1929, *Nature,* **123** , 313.

Schmorak, M. R. 1981, *Nucl. Data Sheets,* **32** , 87.

Schramm, D. N. 1974, *Ann. Rev. Astr. Ap.,* **12** , 303.

Schramm, D. N., and Wasserburg, G. J. 1970, *Ap. J.,* **162** , 57. (SW)

Seeger, P. A., and Schramm, D. N. 1970, *Ap. J.,* **160** L157.

Shirk, J. 1975, Ph.D. thesis, Washington University.

Symbalisty, E. M. D., and Schramm, D. N. 1981, *Rep. Prog. Phys.,* **44** , 293.

Takahashi, K., and Yokoi, K. 1981, *Nucl. Phys.,* **A404** , 578.

Talbot, R. J., and Arnett, W. D. 1973, *Ap. J.,* **186** , 69.

Tatsumoto, M., Unruh, D. M., and Desborough, G. A. 1976, *Geochim. Cosmochim. Acta,* **40** , 617.

Tatsumoto, M., and Shimamura, T. 1980, *Nature,* **286** , 118.

Tatsumoto, M., Shimamura, T., Manuel, O. K., Unruh, D. M., and Pellas, P. 1980, *Lun. Planet. Sci.,* **XI** , Abstracts 1125.

Thielemann, F. -K. 1984, in *Stellar Nucleosynthesis,* ed. C. Chiosi and A. Renzini (Dordrecht: Reidel), p.389.

Thielemann, F. -K., Metzinger, J., and Klapdor, H. V. 1983a, *Z. Phys. A.,* **309** , 301.

__________. 1983b, *Astron. Ap.,* **123** , 162.

Thielemann, F. -K., and Truran, J. W. 1985, in *Nucleosynthesis and Its Implications on Nuclear and Particle Physics,* ed. J. Audouze and N. Mathieu (Dordrecht: Reidel) in press.

Tinsley, B. M. 1975, *Ap. J.,* **198** , 145.

__________. 1977, *Ap. J.,* **216** , 548.

__________. 1980, *Fund. Cosm. Phys.,* **5** , 287.

Walker, F. W., Miller, D. G., and Feiner, F. 1983, *Chart of the Nuclides* (San Jose: General Electric) p. 45.

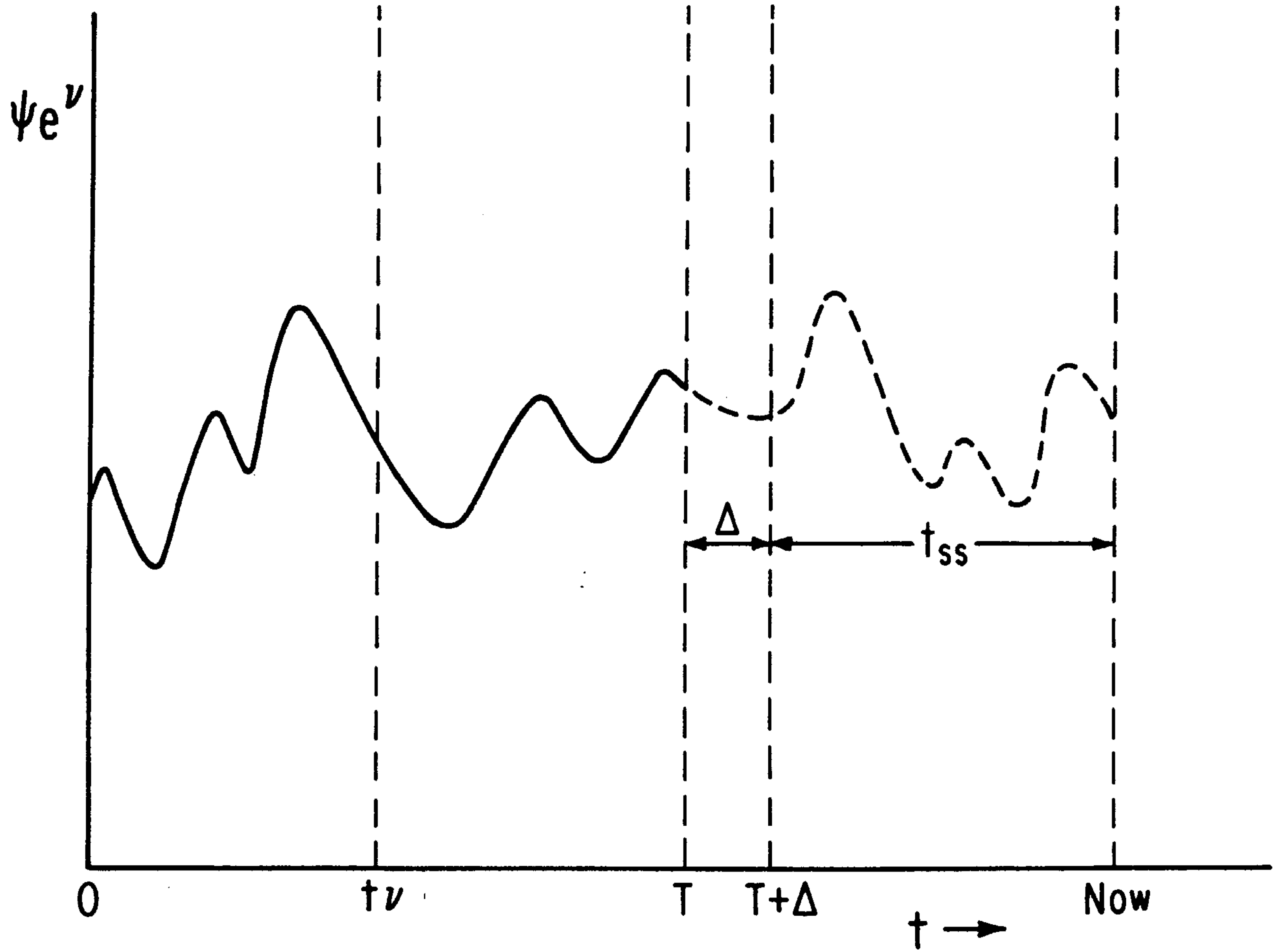
Wasserburg, G. J., Schramm, D. N., and Huneke, J. C. 1969, *Ap. J.,* **157** , L91.

Woosley, S. E., and Fowler, W. A. 1979, *Ap. J.,* **233** , 411.

Yokoi, K., Takahashi, K., and Arnould, M. 1983, *Astr. Ap.,* **117** , 65.

Captions

Figure 1. A schematic diagram showing the effective nucleosynthesis rate ψe^{ν} as a function of time. T is the total duration of nucleosynthesis and t_{ν} is the mean time for the formation of the elements. Δ is the time interval between the end of nucleosynthesis and solidification of solar system bodies. t_{ss} is the age of the solar system solid bodies. The total age of the elements is $T + \Delta + t_{ss}$.


ψe^ν
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